浙江大学 2013-2014 学年 秋冬 学期

《计算理论》课程期末考试试卷

课程号：21120520　开课学院：计算机学院

考试试卷：☑ A卷　☐ B卷

考试形式：☑ 闭卷　☐ 开卷，允许带 ________入场

考试日期: 2014 年 1 月 15 日，考试时间：120 分钟

诚信考试, 沉着应考, 杜绝违纪

考生姓名________________ 学号________________ 所属院系________________

题序	1	2	3	4	5	6	总分
得分							
评卷人							

Zhejiang University
Theory of Computation, Fall-Winter 2013
Final Exam

1. **(24%) Determine whether the following statements are true or false. If it is true fill a ◯ otherwise a × in the bracket before the statement.**

   (a) (　) Language $\{a^m b^n c^j | m, n, j \in \mathbb{N} \text{ and } m + n + j \geq 2014\}$ is regular.

   (b) (　) Let L be a regular language, so is $\{ww^R |\ w \in \Sigma^* \text{ and } w \in L\}$.

   (c) (　) Let L_1 and L_2 be two languages. If $L_1 L_2$ is regular, then either L_1 or L_2 is regular.

   (d) (　) Let L be a context-free language, then L^* is also context-free.

   (e) (　) Language $\{w_1 \# w_2 \# \cdots \# w_n | n \in \mathbb{N},$ for each $i, w_i \in \{a, b\}^*$ and for some i, w_i is a palindrome$\}$ is context-free.

   (f) (　) Let L be a context-free language, then so is $H(L) = \{x | \exists y \in \Sigma^*, |x| = |y| \text{ and } xy \in L\}$.

   (g) (　) Language $\{\text{“}M_1\text{”}\text{“}M_2\text{”} | M_1 \text{ and } M_2 \text{ are FA}, L(M_1) \subseteq L(M_2)\}$ is undecidable.

   (h) (　) There's a function φ such that φ can be computed by some Turing machines, yet φ is not a primitive recursive function.

   (i) (　) If L_1, L_2, and L_3 are all recursively enumerable, then $L_1 \cap (L_2 \cup L_3)$ must be recursively enumerable.

   (j) (　) Let L_1 and L_2 be two recursively enumerable language. If $L_1 \cup L_2$ and $L_1 \cap L_2$ are recursive, then both L_1 and L_2 are recursive.

   (k) (　) Let L be a recursively enumerable language and $L \leq_\tau \overline{H}$, then L is recursive, where $H = \{\text{“}M\text{”}\text{“}w\text{”} |$ Turing machine M halts on $w\}$.

   (l) (　) The set of undecidable languages is uncountable.

2. **(20%)** Decide whether the following languages are regular or not and provide a formal proof for your answer.

   (a) $L_1 = \{uvu^R | u, v \in \{a, b\}^+\}$

   (b) $L_2 = \{uvu | u, v \in \{a, b\}^+\}$

   where $L^+ = LL^*$.

3. **(20%)** Let $L_3 = \{ab^m c^n a^{m+2n} c | m, n \in \mathbb{N}\}$.

   (a) Give a context-free grammar for the language L_3.

   (b) Design a PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ accepting the language L_3.

   Solution: (a)

(b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

	(q, σ, β)	(p, γ)
$K = \{\_\_\_\_\_\_\_\_\}$ $\Sigma = \{a, b, c\}$ $\Gamma = \{\_\_\_\_\_\_\_\_\}$ $s = \_\_\_\_\_\_\_\_$ $F = \{\_\_\_\_\_\_\_\_\}$		

4. **(12%)** Try to construct a Turing Machine to decide the following language

$$L = \{ww^R | w \in \{0, 1\}^*\}.$$

Where w^R is the inverse of w. Always assume that the Turing machines start computation from the configuration $\triangleright \sqcup w$. When describing the Turing machines, you can use the elementary Turing machines described in textbook.

5. **(12%)** Show that the function: $\varphi : \mathbb{N} \to \mathbb{N}$ given by

$$\varphi(x) = \begin{cases} x \mod 3, & \text{if } x \text{ is a composite number;} \\ x^2 + 1, & \text{otherwise.} \end{cases}$$

is primitive recursive.

6. **(12%)** Consider the problem

$L_{even} = \{``M"|M$ is a TM and $L(M)$ contains at least one string of even number of b's$\}$

(a) Show that L_{even} is recursively enumerable.

(b) Show that L_{even} is non-recursive.

Enjoy your Spring Festival!