浙江大学 2006－2007 学年秋冬季学期

《计算理论》课程期末考试试卷

开课学院：计算机学院　　考试形式：闭卷　　允许带＿＿＿＿＿＿入场

考试时间：2007 年 1 月 24 日，　所需时间：120 分钟，任课教师：＿＿＿＿＿

考生姓名：＿＿＿＿＿＿＿学号：＿＿＿＿＿＿＿专业：＿＿＿＿＿＿＿＿

题序	一	二	三	四	五	六	七	八	总 分
得分									
评卷人									

1. (20%) Tell whether the statements below are true (T) or false (F).

(1) (F) If a DFA $\boldsymbol{M}$ contains a self-loop on some state q, then $\boldsymbol{M}$ must accept an infinite language.

(2) (F) $\{w : w$ is a regular expression for $\{a^n b^m : n + m \leq 2007\}\}$ is a finite language.

(3) (F) $\boldsymbol{P}(\{n \in \boldsymbol{N} : n \leq 2007\})$ is uncountable. ($\boldsymbol{P}(\boldsymbol{A})$ is the power set of $\boldsymbol{A}$.)

(4) (F) The concatenation of a regular language and a non-regular language is necessarily non-regular.

(5) (T) Suppose that $\boldsymbol{L}$ is context-free and $\boldsymbol{R}$ is regular, $\boldsymbol{L} - \boldsymbol{R}$ is context-free language.

(6) (T) The nondeterministic Turing machine is not stronger more than the other Turing machines on the computability.

(7) (F) The complement of every recursive enumerable language is necessarily non-recursive enumerable.

(8) (T) The class of recursively enumerable language is closed under union and intersection.

(9) (T) $\varnothing \subseteq \{\varnothing, a\}$.

(10) (T) There is no algorithm that decides, for an arbitrary given Turing machine $\boldsymbol{M}$ and input string w, whether or not $\boldsymbol{M}$ accepts w.

2. (12%) Say whether each of the following languages is regular or non-regular, and prove your answers.

(*a*) $\boldsymbol{L}_1=\{a^{i-j} : i=3j\}$

(*b*) $\boldsymbol{L}_2=\{w: w\in\{a, b\}^* \text{ and } w\neq w^R \}$

Solution :

(*a*) $\boldsymbol{L}_1$ is regular. Because $\boldsymbol{L}_1$ can be rewritten as

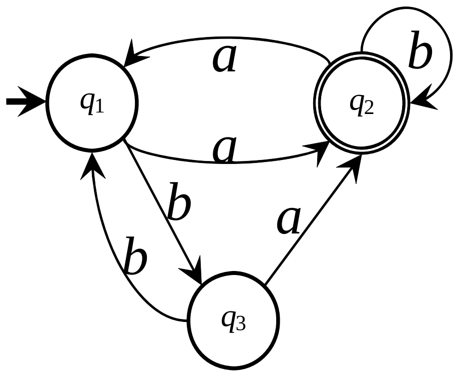
$\boldsymbol{L}_1=\{a^{2n} : n=0,1, \ldots\}$

And its regular expression is $(aa)^*$

(*b*) $\boldsymbol{L}_2$ is not regular.

Suppose $\boldsymbol{L}_2$ is regular language, then the complement of $\boldsymbol{L}_2$ is also regular language. But the complement of $\boldsymbol{L}_2$ is $\{w: w\in\{a, b\}^* \text{ and } w=w^R \}$ that can be proved not regular by using Pumping theorem.

3. (14%) Describe the language accepted by the following finite automaton, and give a Context-free Grammar for the language.



Solution :

(*a*) The language accepted by the given finite automaton is

$L=(aa\cup bb)^*(a\cup ba)b^*((aa)^*\cup(ab^*a)^*)$

(*b*) The same language is generated by the CFG $G=(V, \Sigma, R, S)$, where

$V=\{q_1, q_2, q_3, a, b\}$, $\Sigma=\{a, b\}$, $S= q_1$,

$R=\{$ $q_1 \rightarrow aq_2$,

$q_1 \rightarrow bq_2$,

$q_2 \rightarrow aq_1$,

$q_2 \rightarrow bq_2$,

$q_3 \rightarrow aq_2$,

$q_3 \rightarrow bq_1$,

$q_2 \rightarrow e\}$

4. (18%) Consider the pushdown automaton $\boldsymbol{P}=\{\boldsymbol{K}, \Sigma, \Gamma, \Delta, s, \boldsymbol{F}\}$ where:

$\boldsymbol{K}=\{s, f\}$

$\Sigma=\{a, b\}$

$\Gamma=\{b\}$

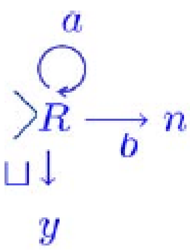
$\boldsymbol{F}=\{f\}$

Δ:
$((s, a, e), (f, e))$
$((s, b, e), (s, b))$
$((s, a, b), (s, b))$
$((s, e, e), (f, e))$
$((f, a, e), (f, e))$
$((f, b, e), (s, b))$

Describe the language accepted by ***P***, and give a Turing machine ***M*** that decides the same language.

Solution :

- The language accepted by ***P*** is $L=\{a^*\}$.
- A Turing machine ***M*** decides the language L is:



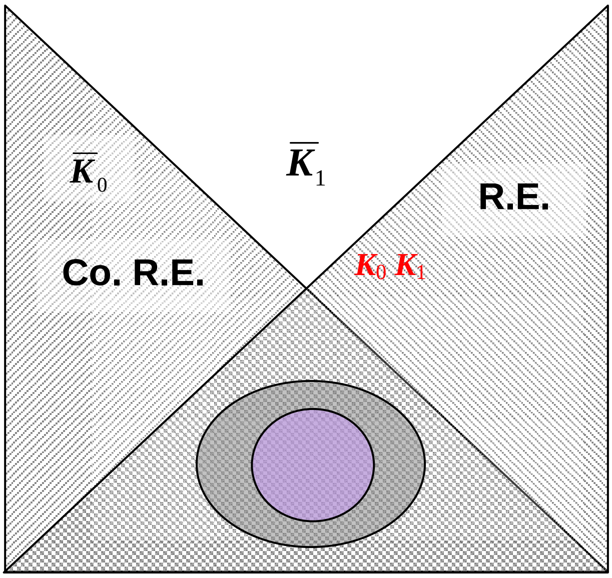
5. (10%) Show the following function that is primitive recursive.

$$f(x)=\begin{cases} x+1, & \text{if } x \text{ is odd} \\ x, & \text{if } x \text{ is even} \end{cases}$$

Solution :

- $f(x)=(x+1)\cdot rem(x, 2)+x\cdot(1\sim rem(x, 2))$

6. (14%) Let $\boldsymbol{K}_0$ = {"***M*** " "***w***" : ***M*** halts on input string ***w*** }, $\boldsymbol{K}_1$ = {"***M*** " : ***M*** halts on input string "***M*** " }. Try to sign the languages of $\overline{\boldsymbol{K}}_0$, $\overline{\boldsymbol{K}}_1$, recursive, context-free, regular to the corresponding zone of the following figure: (Note: Co. R.E. means that the complement of R.E. set is also R.E. .)



7. (12%) Show that the following language:

$\boldsymbol{H}$ = {"***M***" : ***M*** is a Turing Machine and halts on empty string}

is recursively enumerable. An informal description suffices.

Solution :

The universal Turing machine U can semidecides the language

{"***M***" "***w***": ***M*** is a TM and halts on e}