it is true fill a T otherwise a F in the bracket before the statement.

(a) ( ) For any regular languages $L_1 \subseteq L_2 \subseteq \cdots \subseteq L_n \subseteq \cdots$, then the union $\cup_{n=1}^{n=\infty} L_n$ is also regular.

(b) ( ) If A is regular and both of B and $A \cap B$ are non-regular, then $A \cup B$ is non-regular.

(c) ( ) Let $D_{DFA} = \{\text{“}M\text{”} \mid M \text{ is a DFA, “}M\text{”} \notin L(M)\}$, then D_{DFA} is not regular, but recursive, where “M” is the encoding of DFA M, just as Turing Machine.

(d) ( ) The language $\{a^i b^j c^k \mid i, j, k \in \mathbb{N}, \text{ and } k \neq i + j\}$ is context-free.

(e) ( ) A is recursively enumerable and B is regular, then $A \cap B$ is recursive.

(f) ( ) Let A and B be two recursively enumerable language. If both $\overline{A \cup B}$ and $\overline{A \cap B}$ are also recursively enumerable, then A and B are recursive.

(g) ( ) Let $A = \{\text{“}M\text{”} \mid \text{ TM } M \text{ halts on at least 2020 strings }\}$. Suppose $A \leq_\tau \overline{B}$, then B is not recursively enumerable.

(h) ( ) Let A, B, C be arbitrary languages. If $A \leq C$, $B \leq C$ and C is recursively enumerable, then $A \cap B$ is recursively enumerable.

(i) ( ) Let A be a language, if there is a Turing machine M halts on x for every $x \in A$, then A is recursive

(j) ( ) If A is a Turing-enumerable language, then either A is recursive, or $\overline{A}$ is not Turing-enumerable .

(k) ( ) There are some languages in $\mathbb{NP}$ are not recursive.

(l) ( ) There are some languages that cannot be semi-decided by any Turing machine.

2. **(18 pts)** ON FA AND REGULAR LANGUAGES
Say whether each of the following languages is regular or not regular? Prove your answers.

(a) $L_1 = \{uvu^R \mid u \in \{a,b\}^+, \text{ and } v \in \{a,b\}\}$.

(b) $L_2 = \{uvu^R \mid u \in \{a,b\}^+, \text{ and } v \in \{a,b\}^+\}$.

3. **(18 pts)** ON PDA AND CONTEXT-FREE LANGUAGES
Let $L_3 = \{xx^R yy^R \mid x, y \in \{a,b\}^+, |y| \text{ is odd, and } bba \text{ is a substring of } x\}$.

(a) Construct a context-free grammar that generates the language L_3.

(b) Construct a pushdown automata that accepts L_3.

SOLUTION: (a)

(b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

	(q, σ, β)	(p, γ)
$K =$ ______		
$\Sigma = \{a, b\}$		
$\Gamma =$ ______		
$s =$ ______		
$F =$ ______		

4. **(12 pts)** Construct a Turing machine that decides the following language:

$$L_4 = \{ucvcww^R \mid u, v, w \in \{a,b\}^*\}$$

When describing the Turing machines above, you can use the elementary Turing machines described in textbook. Always assume that the Turing machines start computation from the configuration $\triangleright\underline{\sqcup}x$ where $x \in \{a,b,c\}^*$ is the input string.

5. **(10 pts)** ON PRIMITIVE RECURSIVE FUNCTION
Show the following function $\varphi : \mathbb{N} \times \mathbb{N} \mapsto \mathbb{N}$ by

$$\varphi(x,y) = \begin{cases} x/2 + 2y, & \text{if } x \text{ is even, and } y \text{ is a composite number} \\ |x - y|, & \text{otherwise} \end{cases}$$

is primitive recursive.