浙江大学 2017-2018 学年 秋冬 学期

《计算理论》课程期末考试试卷

课程号： 21120520 **开课学院：** 计算机学院

考试试卷： ☑ A卷 ☐ B卷

考试形式： ☑ 闭卷 ☐ 开卷，允许带 _____入场

考试日期: 2018 年1 月24 日，**考试时间：** 120 分钟

诚信考试, 沉着应考, 杜绝违纪

考生姓名_____ **学号**_____ **所属院系**_____

题序	1	2	3	4	5	6	总分
得分							
评卷人							

Zhejiang University
Theory of Computation, Fall-Winter 2017
Final Exam

1. **(24 pts) Determine whether the following statements are true or false. If it is true fill a T otherwise a F in the bracket before the statement.**

   (a) ( ) Let A, B be two languages. If both A and $A \cup B$ are regular, then B is definitely regular.

   (b) ( ) If A is regular and B is non-regular, then $A \circ B$ must be non-regular.

   (c) ( ) Language $\{xcy | x, y \in \{a, b\}^*, |x| \leq |y| \leq 3|x|\}$ is context-free.

   (d) ( ) Every regular language can be generated by a context-free grammar.

   (e) ( ) If A is recursive and $B \subseteq A$, then B is recursive as well.

   (f) ( ) Recursively enumerable languages are always infinite.

   (g) ( ) There's a function φ such that φ can be computed by some Turing machines, yet φ is not a primitive recursive function.

   (h) ( ) Let A and B be recursively enumerable languages and $A \cap B = \emptyset$. If $\overline{A \cup B}$ is also recursively enumerable, then both A and B is decidable.

   (i) ( ) The language $\{$"M" "w"| TM M accepts w in less than 2018 steps $\}$ is recursive.

   (j) ( ) The language $\{$"M" | TM M accepts exactly 2018 strings$\}$ is recursively enumerable but not recursive.

   (k) ( ) Let $H_e = \{$"M"| TM M halts on $e\}$. If $H_e \leq \overline{L}$, then L is recursive enumerable but not recursive.

   (l) ( ) A language L is recursive if and only if it is Turing-enumerable.

2. **(18 pts)** On FA and Regular Languages
   Say whether each of the following languages is *regular* or *not regular*? Prove your answers, where $\{a,b\}^+ = \{a,b\}^* \circ \{a,b\}$.

   (a) $L_1 = \{wtw | w,t \in \{a,b\}^+\}$

   (b) $L_2 = \{wtw | w,t \in \{a,b\}^*\}$

3. **(20 pts)** On PDA and Context-Free Languages
   Let $L_3 = \{ww^R ca^m b^n | w \in \{a,b\}^*, m,n \in \mathbb{N}, m \neq n\}$.

   (a) Construct a context-free grammar that generates the language L_3.

   (b) Construct a pushdown automata that accepts L_3.

   Solution:

   (a)

(b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

	(q, σ, β)	(p, γ)
$K =$ _____		
$\Sigma = \{a, b, c\}$		
$\Gamma =$ _____		
$s =$ _____		
$F =$ _____		

4. **(10 pts)** On Turing Machine
Design a single tape Turing machine M that decides the language L_4

$$L_4 = \{uvcww^R | u, v, w \in \{a, b\}^*, |u| = 2|v|\}$$

When describing the Turing machines above, you can use the elementary Turing machines described in textbook. Always assume that your Turing machine starts from the configuration $\triangleright\underline{\sqcup}x$, where x is the input string.

5. **(10 pts)** On Primitive Recursive Function
Show the following function $\varphi_k : \underbrace{\mathbb{N} \times \mathbb{N} \times \cdots \times \mathbb{N}}_{k} \mapsto \mathbb{N}$, and $k \in \mathbb{N}$, $k \geq 2$

$$\varphi_k(n_1, n_2, \cdots, n_k) = \max_k\{n_1, n_2, \cdots, n_k\}$$

is primitive recursive.

6. **(18 pts)** On Undecidability
   Classify whether each of the following languages are recursive, recursively enumerable but not recursive, or non-recursively enumerable. Prove your answers, but you may not simply appeal to Rice's theorem.

   (a) $L_5 = \{\text{“}M\text{”} | M \text{ is a TM, and } L(M) \text{ is uncountable}\}$

   (b) $L_6 = \{\text{“}M\text{”} | \text{ TM } M \text{ accepts at least two strings of different lengths}\}$