计算理论习题集

2022 年 12 月 3 日

以下习题主要来自于本校计算理论历年试卷, 解答来自于标准答案, 我收集到的答案以及我自己写的答案. 为保持一致, 题目基本为英文. 如有错误, 欢迎指正!

1 Finite Automata and Regular Language

1. Determine whether the following statements are true or false.

    (1) Infinite unions of regular sets are regular.

    (2) Language $\{a^{6n}b^{3m}c^{p+10} \mid n \geq 0, m \geq 0, p \geq 0\}$ is regular.

    (3) If L_1 and $L_1 \cup L_2$ are regular languages, then L_2 is a regular language.

    (4) Let A, B, C be three languages, and $A \subseteq B \subseteq C$. If both A and C are regular, then B is regular.

    (5) If A is regular and B is non-regular, then $A \circ B$ must be non-regular.

    (6) If A is non-regular and both B and $A \cap B$ are regular, then $A \cup B$ is non-regular.

    (7) Language $\left\{a^ib^jc^k \mid i, j, k \in \mathbb{N} \text{ and } i + j \not\equiv k \mod 3\right\}$ is not regular.

    (8) Let A and B be two regular languages, then $A \oplus B$ is also regular.

    (9) $\{w : w \text{ is a regular expression for } \{a^nb^m : n + m \leq 2007\}\}$ is a finite language.

    (10) If $L_1 \circ L_2$ is a regular language, then either L_1 or L_2 is regular.

2. 写出以 ab 串结尾的语言 (字母表为 $\{a, b\}$) 的正则表达式, 画出 NFA, 转化成 DFA, 并得到最小化 DFA.

3. Say whether each of the following languages is regular or not (prove your answers):

    (1) $L_1 = \{w \mid w \in \{a, b\}^* \text{ and } w \neq w^R\}$.

    (2) $L_2 = \{wtw \mid w, t \in \{a, b\}^+\}$.

    (3) $L_3 = \{wtw \mid w, t \in \{a, b\}^*\}$.

    (4) $L_4 = \left\{uvu^R \mid u, v \in \{a, b\}^+\right\}$.

2 Context Free Language

1. Determine whether the following statements are true or false.

   (1) Suppose that L is context-free and R is regular, then $L - R$ is context-free language.

   (2) Every regular language can be generated by context-free grammar.

   (3) A and B are two context-free languages, so is $A \oplus B$, where $A \oplus B = (A - B) \cup (B - A)$.

   (4) Let L be a context-free language, then so is $H(L) = \{x \mid \exists y \in \sum^*, |x| = |y| \text{ and } xy \in L\}$.

   (5) Language$\{xcy \mid x, y \in \{a,b\}^*, |x| \leq |y| \leq 3|x|\}$ is context-free.

2. Let $L = \{ab^m c^n a^{m+2n} c \mid m, n \in \mathbb{N}\}$.

   (1) Give a context-free grammar for the language L.

   (2) Design a PDA $M = (K, \sum, \Gamma, \Delta, s, F)$ accepts the language.

3. 令 $L = \{w \in \{a,b\}^* \mid a \neq b\}$, 即那些 a, b 个数不相等的串构成的语言. 试用 CFG 写出能表示 L 的文法.

3 Turing Machine and Undecidability

1. (1) If A is recursive and $B \subseteq A$, Then B is recursive as well.

   (2) There's a function φ such that φ can be computed by some Turing machines, yet φ is not a primitive recursive function.

   (3) If L_1, L_2, and L_3 are all recursively enumerable, then $L_1 \cap (L_2 \cup L_3)$ must be recursively enumerable.

   (4) Let L_1 and L_2 be two recursively enumerable languages. If $L_1 \cup L_2$ and $L_1 \cap L_2$ are recursive, then both L_1 and L_2 are recursive.

   (5) Let A and B be recursively enumerable languages and $A \cap B = \emptyset$. If $\overline{A \cup B}$ is also recursively enumerable, then both A and B is decidable.

   (6) Let L be a recursively enumerable language and $L \leq_\tau \bar{H}$, then L is recursive, where $H = \{``M'' ``w'' \mid \text{Turing machine } M \text{ halts on } w\}$.

   (7) The set of undecidable languages is uncountable.

2. Try to construct a Turing Machine to decide the following language.

$$L = \left\{ww^R \mid w \in \{0,1\}^*\right\}.$$

   You can assume the start configuration of the Turing machine is $\triangleright \sqcup w$.

3. Show that the function: $\varphi : \mathbb{N} \to \mathbb{N}$ given by

$$\varphi(x) = \begin{cases} x \bmod 3, & \text{if } x \text{ is a composite number;} \\ x^2 + 1, & \text{otherwise.} \end{cases}$$

4. Show the following function $\varphi_k : \underbrace{\mathbb{N} \times \mathbb{N} \times \cdots \times \mathbb{N}}_{k} \mapsto \mathbb{N}$, and $k \in \mathbb{N}, k \geq 2$

$$\varphi_k \left(n_1, n_2, \cdots, n_k\right) = \max_k \left\{n_1, n_2, \cdots, n_k\right\}$$

is primitive recursive.

5. $L_{\text{even}} = \{``M'' \mid M$ is a TM and $L(M)$ contains at least one string of even number of b' s $\}$

(1) Show that L_{even} is recursively enumerable.

(2) Show that L_{even} is non-recursive.

6. Classify whether each of the following languages are recursive, recursively enumerable but not recursive, or non-recursively enumerable.

   1. The language $AL = \{``M'' \mid$ TM M accepts at least 2018 strings $\}$.
   2. The language $E = \{``M'' \mid$ TM M accepts exactly 2018 strings $\}$.
   3. The language $AM = \{``M'' \mid$ TM M accepts at most 2018 strings $\}$.